

Problem Set 7

This week the assignment will be in two parts with instructions as follows:

Part 1: Read each problem. Write a sentence or two about the approach you might take to solve each problem. Draw a picture to illustrate the scenario. Write a formula that might be needed to help set up or solve the problem. Or interpret what each is asking for using mathematical notation.

Part 2: You do not need to attempt these problems prior to class. These problems will be attempted with your group during our class on Monday.

Part 1:

1. Consider the following transformation of a logarithm of base $b > 1$.

$$y = \log_b(Ax + B) + C$$

where A , B , and C are positive constants.

(A) Determine the exact values of all intercepts and asymptotes.

(B) Sketch an accurate graph of $y = \log_b(Ax + B) + C$, labeling all intercept(s) and asymptote(s).

(C) Find the inverse of $y = \log_b(Ax + B) + C$.

(D) Determine the exact values of all intercepts and asymptotes of the inverse function.

(E) Sketch an accurate graph of the inverse, labeling all intercept(s) and asymptote(s).

2. Consider three possible savings accounts. One offers an interest rate of 2.5% compounded annually, another offers 2.4% compounded quarterly, and the third offers 2.1% compounded continuously. Answer the following questions.
- (A) What is the relative/continuous growth rate for each investment? (the r value in the continuous model $y = Pe^{rt}$)
- (B) What is the effective (actual) percentage growth annually for each investment? (the r value in the periodic model $y = Pb^t$, where $b = 1 + r$)
- (C) What is the doubling time for each investment? (Round to the nearest month)
3. You just brought a new \$50,000 Porsche with the loan you were supposed to use for your tuition.
- (A) Sadly you learn that this car loses 15% of its value every year. Write an equation to represent the value of your car as a function of time since you purchased it.
- (B) Suppose your insurance company calculates its premiums based on the value of a car. Instead of using your formula in part A, they assume that the value of the car will depreciate by \$4,000 every year. Write an equation to represent their value of your car as a function of time since you purchased it.
- (C) Sketch a graph of both equations (including your window). Will the insurance company be over or under charging during the first few years?

4. In the past decade, alarm has spread over several parts of our country concerning radon, a highly radioactive gas, which can have severe deleterious effects on people who remain in contact with it for prolonged periods. This gas, which is formed by the disintegration of radium, is found in the soil over most of the earth. It can seep through foundations of homes and office buildings and since it is colorless and odorless, detection by the natural senses is impossible. Radon detection kits are available at most hardware stores. The chief use of radon is in the treatment of cancer by radiotherapy. F.O. Dorn discovered one of the most common forms of radon, radon 222, in 1900. He called it radium emanation. We shall investigate this form of radon.

Suppose radon is detected at a local elementary school on Monday, October 5 at 9 a.m. Students and personnel are immediately moved to a different location, and steps are taken immediately so that no additional radon contaminates the area.

Let t be the time (in days) after October 5 at 9 a.m. and let Q be the amount of radon present at the school at time t . The following table shows the amount of radon present at various times. Since this is radioactive the radon decays exponentially.

t days	1.2	2.6	3.1	4.1	8.5
Q	40.15	31.15	28.44	23.71	10.64

- (A) Write a formula for the function that best models the data.
- (B) By what percentage does the radon decrease each day?
- (C) How much radon is detectable at 9 a.m. on October 6?
- (D) Radon levels of less than four units are considered safe. How many days will it take for this site to be considered safe?
- (E) Suppose that the most sensitive current technology can only measure levels of .01 units of radon in a given area. In how many days will the radon on this site be undetectable (nearest day and hour)?
- (F) What is the half-life of radon?
- (G) Will the amount of radon on this site theoretically ever reach zero? Practically? Explain.

Part 2

5. Consider the following population scenarios. In each, determine if an exponential, linear, or neither of these models would be a suitable fit. Explain your choice. If exponential or linear, find an equation to best describe the population as a function of time measured in years, assuming at $t = 0$ the population is 500,000.

a) each year, the town grows by roughly 1000 residents.

b) each year, the town grows by roughly 9%.

c) each year, the town is decreasing at a continuous rate of 4%.

d) each year, the town shrinks by roughly 15%.

e) each year, the town loses roughly 2500 residents.

6. Write the following expressions with no terms in the exponent and no negative exponents.

a) 2^{x+3}

b) 3^{2x-1}

c) $\frac{1}{3^{x-3}}$

d) $2(5^{-x+2})$

7. Solve the following exactly:

(A) $\log_2(1-x) + \log_2(4-x) = 3$

(B) $e^{2x} - 4e^x = 5$

(C) $18^x = 3^{2x-1}$

(D) $2\log_4(3x+1) + 4 = 9$

(E) $\log(x) - 2\log(x-3) = 1$